KINETIC TRANSITIONS IN MEDIA WITH MICROCRACKS AND METAL FRACTURE IN STRESS WAVES

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UDC 539.42:620.172.254

At this time, microcracks have received the designation of a legitimate defect in the structure of solids. Investigations performed in metals by microscopic methods show that the deformation under different kinds of loading (developed plastic deformation, creep, dynamic and fatigue loading) is accompanied by the multiple generation and growth of microcracks [1-4]. These latter are characterized by a size distribution, possess anisogeometric shape (greater for brittle bodies and smaller for plastic), fracture and relaxation properties are determined substantially by the kinetics of microcrack growth [5]. An experimental study of fracture under shock loads also established multiple microcrack generation and growth in stress waves [3, 4, 6, 7]. A fractographic study shows that dynamic fracture includes the following main stages as a rule: rapid microcrack generation, their growth under the effect of tensile stresses, microcrack merger and material separation with the formation of one or more free surfaces. Starting from the characteristic microcrack shape in [3, 4, 6], fracture under shockwave loads is subdivided into two classes as for quasistatic loads: viscous and brittle. The question of the correlation of dynamic and quasistatic data characterizing fracture as a microcrack accumulation process is discussed in [3, 8-10].

This aspect of fracture apparently merits a detailed study since the observable commonality of the microcrack generation and growth processes in substantially different loading modes permits association of a state variable, an independent thermodynamic coordinate, with the microcracks and its consideration as a universal structural parameter. Models known to this time for the fracture process that take account of microcrack generation and growth, consider it from phenomenological aspects as a rule. However, as is noted in [11, 12], underlying the description of the deformation and fracture of media with defects of this kind should be a statistical model of defect accumulation, including the investigation of the whole ensemble of microcracks, each of which is developed in a random stress field determined by the microcracks of the environment and the microinhomogeneity of the material. Experiments show that fracture seeds in the form of some kind of imperfections exist in every real solid. Consequently, the growth of fracture from already existing defects must be examined in the construction of a fracture model. It is also characteristic that the physical regularities of the fracture process are determined by local relationships, which change substantially even for constancy of the initial values.

1. A statistical-thermodynamic model of solids with microcracks is proposed in [13, 14]. A symmetric tensor p_{ik}, the microcrack density tensor, is used as an additional state variable to characterize the bulk microcrack concentration and their preferred orientation. This latter is determined by taking the average over the statistical ensemble of microcracks

$$p_{ik} = n \int s_{ik} W ds d^3\nu \qquad (1.1)$$

where $s_{ik} = s\nu_i\nu_k$ is a symmetric tensor governing the volume s and the orientation ν of the microcracks of "normal" separation; $W = Z^{-1}\exp(-E/T)$ is the microcrack Gibbs distribution law by size and orientation; Z is a normalizing factor; E is a microcrack energy; T is the temperature measured in energetic units; and n is the number of microcracks per unit volume ($n \approx 10^{11}$-10^{13}).

A study of the properties of an elastic medium with microcracks on the basis of (1.1) for uniaxial tension of the specimen permitted clarification of the characteristic reactions of solids to crack formation [14]. Displayed in Fig. 1a are dependences of the parameter p_{zz} on the stress σ_{zz} at a fixed temperature T for different values of the structural parameter δ (curves 1-3 correspond to $\delta < \delta_c$, $\delta_c < \delta < \delta_*$, $\delta > \delta_*$). It is shown in [15] that the

Perm'. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 163-171, January-February, 1987. Original article submitted July 25, 1985.

0021-8944/87/2801-0157$12.50

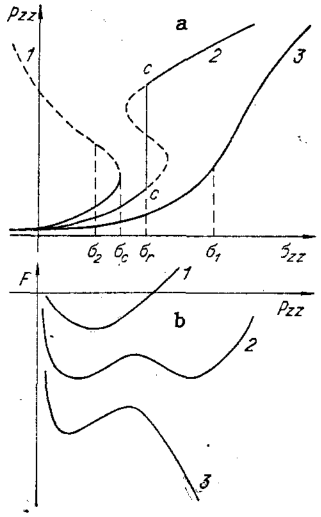

Fig. 1

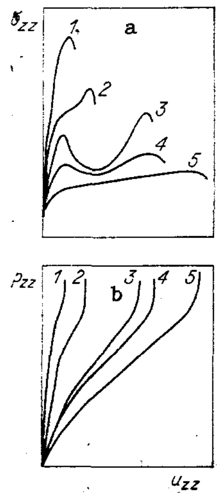

Fig. 2

magnitude of this last depends on the natural scale characteristics of the material: the mean size of the structual heterogeneity and the correlation radius of the microstress fields induced by the microcracks. For values $\delta > \delta_*$, the dependence $p_{zz}(\sigma_{zz})$ is monotonic in nature: a unique microcrack concentration corresponds to the applied stress and the reaction to crack formation is reversible. This result is confirmed by experiments [5, 16]. Metastability in the parameter p_{zz} associated with the orientational degrees of microcrack freedom is observed in the interval $\delta_c < \delta < \delta_*$. An abrupt change in the bulk microcrack concentration occurs here in the ambiguity domain. Such a nature of the change in material density during deformation was first detected by Bauschinger and later confirmed by a significant number of experiments [17]. For values $\delta < \delta_c$, the jump in the parameter p_{zz} will be infinite and the reaction of the material to the microcrack generation and growth in the domain $\sigma_{zz} > \sigma_c$ becomes absolutely unstable. An illustration of the curves $p_{zz}(\sigma_{zz})$ is presented in Fig. 1b, where dependences of part of the free energy associated with microcracks $F = -nT\cdot \ln Z$ corresponding to characteristic reactions are shown (the dependences 1-3 correspond to $\delta > \delta_*$ for $\sigma = \sigma_1$, $\delta_c < \delta < \delta_*$ for $\sigma = \sigma_r$, $\delta < \delta_c$ for $\sigma = \sigma_2$). For values $\delta > \delta_*$, there is one minimum on the curves; the phase metastability in the overlap domain ($\delta_c < \delta < \delta_*$) is associated with the existence of two minimums of the function $F(p_{zz})$. For $\delta < \delta_c$, a metastability domain corresponds to the set of stress values smaller than σ_c; however, the true minimum of the function $F(p_{zz})$ becomes infinitely deep and the bulk microcrack concentration for a finite stress can be arbitrarily high.

The interrelation of the kinetics of crack formation and plastic deformation is studied in [18] in a local equilibrium approximation. The dissipative function for the system in which plastic relaxation and disperse fracture exist has the form

$$TP_S = -\frac{q_k}{T}\frac{\partial T}{\partial x_k} + \sigma'_{ik}e^{p'}_{ik} + \sigma e^p - \Pi'_{ik}\frac{\Delta p'_{ik}}{\Delta t} - \Pi\dot{p} \geqslant 0 \quad (i, k = 1, 2, 3), \tag{1.2}$$

where P_S is the entropy production; q_k, heat flux vector components; $\Pi_{ik} = \partial F/\partial p_{ik}$, thermodynamic force acting on the system when the value of p_{ik} differs from the equilibrium value; σ_{ik}', $e_{ik}^{p'}$, p_{ik}', Π_{ik}' and σ, e^p, p, and Π, traceless and isotropic stress tensor components, plastic strain rates, microcrack density parameter, and tensor Π_{ik}; $(\Delta p_{ik}'/\Delta t) = (dp_{ik}'/dt) - \omega_{i\ell}p_{\ell k}' - \omega_{\ell k}p_{i\ell}'$, tensor derivative with respect to time (the Jaumann derivative [19]); $\omega_{i\ell}$, antisymmetric vortex tensor.

According to (1.2), the constitutive equations are written in a linear approximation in the nonequilibrium as [13, 20]

$$q_i = \lambda_{ik}(p_{\alpha\beta})\frac{\partial T}{\partial x_k}; \tag{1.3}$$

$$\sigma = \zeta(p)e^p - \alpha(p)\dot{p}; \tag{1.4}$$

$$\Pi = \alpha(p)e^p - \beta(p)\dot{p}; \tag{1.5}$$

$$\sigma'_{ik} = L^{(1)}_{iklm}(p_{\alpha\beta})\, e^{p'}_{lm} - L^{(2)}_{iklm}\frac{\Delta p'_{lm}}{\Delta t}; \tag{1.6}$$

$$\Pi'_{ik} = L^{(2)}_{iklm}(p_{\alpha\beta})\, e^{p'}_{lm} - L^{(3)}_{iklm}\frac{\Delta p'_{lm}}{\Delta t} \tag{1.7}$$

with symmetry of the coefficients α, $L_{ik\ell m}^{(2)}$ taken into account and under positive-definiteness conditions for the coefficients λ_{ik}, $L_{ik\ell m}^{(1)}$, and $L_{ik\ell m}^{(3)}$.

Equations (1.3)-(1.7) are quasilinear: the kinetic coefficients λ_{ik}, ζ, α, β, and $L_{ik\ell m}^{(\nu)}$ depend on p_{ik} in the general case. The anisotropy of the kinetic coefficients associated with the structural parameter p_{ik} describes the deformation anisotropy of the mechanical properties and the appearance of texture in the plastically deformed material. Taking account of the symmetry of the tensor p_{ik}, the general form of the dependence of the kinetic coefficients $L_{ik\ell m}^{(\nu)}$ on p_{ik} is the following:

$$L^{(\nu)}_{iklm} = l^{(\nu)}\delta_{il}\delta_{km} + l_1^{(\nu)}(p_{il}\delta_{km} + p_{kl}\delta_{im}) + l_2^{(\nu)} p_{ik} p_{lm} \tag{1.8}$$

($\ell^{(\nu)}$, $\ell_1^{(\nu)}$, and $\ell_2^{(\nu)}$ are phenomenological coefficients). Analogously, we have the following representation for the tensor of the heat-conduction coefficients λ_{ik} coaxial with the tensor p_{ik}:

$$\lambda_{ik} = \lambda_0 \delta_{ik} + \lambda_1 p_{ik},$$

where λ_0 is the heat-conduction coefficient of the initially isotropic material, and λ_1 is the material parameter dependent on the invariants of the tensor p_{ik} in the general case.

The material equations of state include relationships of relaxation type for the stress tensor (1.4), (1.6) and equations of motion for the parameter p_{ik} (1.5) and (1.7). Taken into account in these equations are "crossover" effects: the influence of crack-formation on relaxation processes and plasticity on growth kinetics p_{ik}. Later, the case is considered when the plastic deformations are subject to the condition $\mathrm{Sp}\, e_{ik}^{p} = 0$, while the mean stress σ is determined by the elastic components of the strain tensor

$$u^{e}_{ik} = \frac{1}{2\mu}(\sigma_{ik} - \sigma\delta_{ik}) + \frac{1}{3K}\sigma\delta_{ik} \tag{1.9}$$

(μ and K are the shear and volume compression moduli).

The equations of state (1.4)-(1.7) were used in [18] to describe the regularities of the deformation and fracture under conditions of creep and tension with a constant strain rate $\dot{u}_{zz}$.

A qualitative comparison is made in Fig. 2 of the characteristic deformation curves and the kinetic dependences reflecting the change in p_{ik}. For high strain rates the dependence $\sigma_{zz}(u_{zz})$ is almost linear in nature (curve 1). An abrupt reduction in the resistance to deformation starting with certain values of p_{zz} is associated with the intensive growth in the microcrack bulk concentration during the passage to the absolutely unstable branch of the dependence $p_{zz}(\sigma_{zz})$ (see Fig. 1a, curve 1). The process of microcrack accumulation is developed in the explosive instability regime in this case. As the strain rate diminishes, the linear section shifts to the plasticity section with nonlinear hardening (curve 2), which goes over into the descending branch, as in the first case, corresponding to avalanchelike growth of the microcracks. Unstable deformation (curve 4), accompanied by the appearance of a "tooth" and a flow area is observed in a certain strain rate range.

The appearance of instability is related to the presence of a metastability domain in the parameter p_{zz}. Under conditions of passage from the lower to the upper branch in the metastability domain (see Fig. 1a, curve 2), the relaxation tempo increases abruptly. At relatively moderate strain rates, the passage from the lower to the upper branch occurs almost on the equilibrium transition line c—c and the flow "tooth" is not observed in practice (curve 5). As the strain rate increases, deeper penetration occurs into the metastability domain, resulting in growth of the magnitude of the jump in the stress with an effective increase in the elasticity limit (curve 3). The hardening section here also passes higher, which is explained by the growth of the viscous resistance to deformation.

Therefore, the proposed system of equations permits description of the different solid-body reaction to deformation with the nonlinear kinetics in the microcrack density parameter p_{ik} taken into account.

2. Let us consider formulation of the problem of cleavage during propagation of a plane one-dimensional compression wave in the z direction in a plate. In this case, $e_{xx}^p = e_{yy}^p = \dot{u}_{xx}^e = \dot{u}_{yy}^e = 0$, $p_{xx} = p_{yy} = 0$. We limit ourselves to the first terms in the expansion in (1.8), and then taking account of the assumptions introduced, the system of equations of state in conjunction with the mass and momentum conservation laws takes the form

$$\dot{p} = -\frac{1}{\beta}\Pi; \tag{2.1}$$

$$\sigma'_{zz} = l^{(1)} e_{zz}^{p'} - l^{(2)} \frac{\partial p'_{zz}}{\partial t}; \tag{2.2}$$

$$\Pi'_{zz} = l^{(2)} e_{zz}^{p'} - l^{(3)} \frac{\partial p'_{zz}}{\partial t}; \tag{2.3}$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial z}(\rho v_z); \tag{2.4}$$

$$\frac{\partial}{\partial t}(\rho v_z) = -\frac{\partial}{\partial z}(\rho v_z^2 - \sigma_{zz}), \tag{2.5}$$

where ρ is the material density and v_z is the velocity vector component.

The equations presented must be supplemented also by kinematic relationships connected to the irreversible (plastic) strain rate tensor e_{ik}^p to the elastic rates u_{ik}^e and the complete deformation u_{ik}. Analysis of the deformation in the plate collision problem [21] showed that the strain tensor components do not exceed 0.01 under cleavage fracture conditions for metals. For such values the kinetic relation is

$$\dot{u}_{ik} = e_{ik}^p + \dot{u}_{ik}^e. \tag{2.6}$$

Transforming (2.1)-(2.5) with the relationships (1.9) and (2.6) taken into account after insertion of the parameters $\tilde{\sigma} = \sigma_{zz}/\rho c_l^2$, $\tilde{t} = t/\tau_l$ ($\tau_l = h/c_l$, $c_l = \sqrt{(K + {}^4/_3\mu)/\rho}$ is the longitudinal wave velocity), $\tilde{v} = v_z/c_l$, $\tilde{\rho} = \rho/\rho_0$, $\xi = (h/z)^{-1}$ (h is the plate thickness), $\Pi' = \frac{1}{l^{(3)}}\frac{h}{c_l}\Pi'_{zz}$, $\Pi^0 = \frac{h}{\beta c_l}\Pi$, $\tau_m = l^{(1)}/\mu$, $\varkappa = \frac{2}{3}\frac{2\mu^2}{(3K+2\mu)\rho c_l^2}$, $\alpha = l^{(2)}/l^{(1)}$ the equations are transformed (the symbol ~ of dimensionless variables is later omitted):

$$\frac{\partial p}{\partial t} = -\Pi^0, \ \frac{\partial p'}{\partial t} = \frac{2}{3}\alpha\left(\frac{\partial v}{\partial \xi} - \frac{1}{\varkappa}\frac{\partial \sigma}{\partial t}\right) - \Pi', \tag{2.7}$$

$$\frac{\partial \sigma}{\partial t} = \varkappa\frac{\partial \sigma}{\partial \xi} - \frac{\tau_l}{\tau_m}\sigma - \alpha\varkappa\frac{\partial p'}{\partial t}, \ \frac{\partial}{\partial t}(\rho v) = -\frac{\partial}{\partial \xi}(\rho v^2 - \sigma), \ \frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial \xi}(\rho v).$$

Now find the numerical solution of the system (2.7) that satisfies the boundary

$$\sigma(0,\ t) = \sigma_0(t),\ \sigma(1,\ t) = 0$$

and initial conditions

$$v(\xi,\ 0) = \sigma(\xi,\ 0) = p(\xi,\ 0) = 0,\ \rho(\xi,\ 0) = 1.$$

Here $\sigma_0(t)$ is a given function, $\sigma = \sigma_0(t)$ for $t \le t_1$, and $\sigma_0(t) \equiv 0$ for $t > t_1$. To obtain the finite-difference analog of the mentioned equations, an explicit difference scheme of second-order accuracy is used [22]. The time and space mesh parameters were selected in conformity with the stability condition $\Delta t \le \min\{L_f \Delta x/a\}$, where $L_f \approx {}^1/_3$ is the Courant number, and a is the local speed of sound. The functions Π' and Π^0, represented in terms of statistical integrals in [15], were approximated by the finite expressions

$$\Pi' = -A\sigma\exp(-p_a/p') + B(p' - p_b),$$

$$\Pi^0 = \begin{cases} -L\sigma^2\exp(p) & \text{for } \sigma \geqslant 0, \\ 0 & \text{for } \sigma < 0 \end{cases}$$

(A, B, p_a, p_b, and L are approximation parameters).

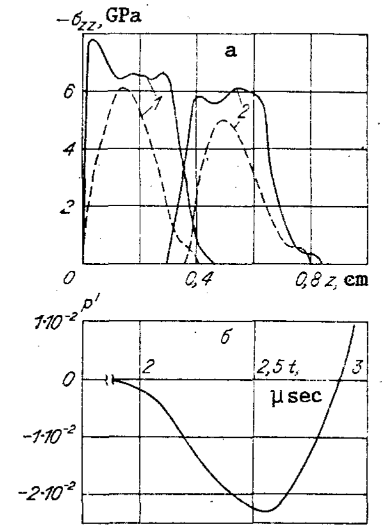

Fig. 3

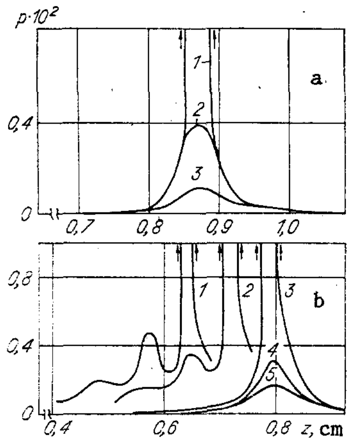

Fig. 4

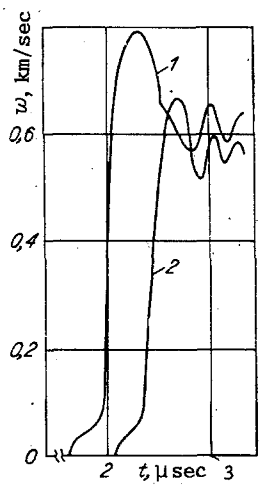

Fig. 5

Certain specific expressions for the kinetic equations describing the growth of microcrack concentration are analyzed in [23] and it is noted that the exponential nature of the dependence of the rate of microcrack accumulation on p permits a qualitatively true pattern to be obtained for the fracture under cleavage conditions.

The coefficients in the equations of state were determined from data on creep experiments for aluminum [24, 25] and were assumed to equal

$$\varkappa = 2{,}5,\ \alpha = 0{,}5,\ \tau_m = 3\cdot 10^{-6}\ \text{sec},\ \tau_l = 1{,}96\cdot 10^{-6}\ \text{sec},$$
$$A = 3,\ B = 0{,}45,\ p_a = 3\cdot 10^{-3},\ p_b = 10^{-3},\ L = 7\cdot 10^{-13}.$$

It is noted in [23] that it is necessary to match the results of cleavage fracture investigations and tests on longevity under load to single fracture kinetics. The introduction of a parameter with the sense of a state variable characterizing the change in material structure, the defectiveness, say, apparently assumes the confirmation of the possibility of describing experimental results in a sufficiently broad range of variation of experiment conditions.

The shape of the load pulse was assumed triangular and rectangular with the duration t_1 = 0.75 μsec and negative stress amplitudes σ_0 = 7 and 8 GPa, respectively.

Presented in Fig. 3 are results of a numerical modeling of the stress-wave propagation and the changes in the traceless component of the microcrack density parameter p_{zz} in the cleavage section with time (t = 0.75 and 1.5 μsec in curves 1 and 2; the solid line is the rectangular pulse and the dashes the triangular pulse).

In the stress domain corresponding approximately to the dynamic yield point, a kinetic transition in the parameter p' is realized, which results in an abrupt increase in the stress relaxation tempo, a change in the plastic wave profile, and the extraction of the elastic predecessor. It is noted in [26] that the shock wave structure is determined mainly by the dependence of the relaxation time on the parameters characterizing the medium and varying during loading. The singularity of the description proposed is that an abrupt change in the parameter p_{ik}' resulting in substantial diminution in the stress relaxation time is realized in a situation analogous to a phase transition of the first kind. A similar dynamic reaction of solids is observed in polymorphic transformations, transitions of solids from one crystalline modification to another [27]. In this case, two shocks are propagated in the substance, one after the other. Splitting of the wave is associated with the anomalous behavior of the shock adiabat of the substance in the phase transition domain, whose analog is an abrupt change in the microcrack orientation mode, resulting in a jump in the magnitude of the deformation [14].

Therefore, by using the tensor damage parameter and taking account of the nonlinear kinetics of its change during deformation, the elastic viscoplastic behavior can be described, including even the unstable plastic reaction, within the framework of the relaxation equation for the stress tensor. A model is proposed in [26] that takes account of the change in the relaxation time, and the structure of wave fronts is studied on its basis. The change in relaxation time is described with the use of activation relationships that take account of the level of the acting stresses. In substance this corresponds to the assumption that the relaxation time in the parameter p_{ik}, the characteristic time of equilibrium buildup for structural changes in the material, is substantially less than the stress relaxation time. This assumption is not made in the description under consideration.

Reflection of a compression wave from a free surface results in the formation of a tensile pulse; the appearance of volume changes in the material because of microcrack growth and fracture.

The kinetics of the growth of the bulk microcrack concentration is represented in Fig. 4 [a for 3.065, 3.0202, and 2.975 μsec (lines 1-3), σ_0 = 4 GPa; and b for 3.4710, 3.3352, 2.9295, 2.9287, and 2.828 μsec (lines 1-5), σ_0 = 7 GPa]; it reflects the following stages of disperse fracture. In the domain of values of the phenomenological parameters corresponding to $\delta > \delta_c$, the microcrack growth process is characterized by their relative weak-volume interaction and disperse fracture is felt mainly as a change in the relaxation properties of the material. As already noted, this is observed especially strongly for an orientation transition that is accompanied by the appearance of a more ordered system of microcracks. An abrupt transition to an ordered structure in macroscopic volumes can result in deformation property anomalies known in dynamics problems as fracture due to plastic shear instability [28]. As the microcrack bulk concentration grows, a transition occurs to the absolutely unstable branch of the dependence $p_{zz}(\sigma_{zz})$ (see Fig. 1a), and the fracture process continuing to remain disperse discovers new features intrinsic to nonlinear systems under kinetic transition conditions [29]; in the domain $\delta < \delta_c$ the microcrack generation and growth process is characterized by explosive instability.

Transition through the explosive instability threshold ($\delta = \delta_c$) is accompanied by replacement of the time asymptotic for p_{zz} and intense growth of defects in the overstress fields generated by the microcracks. The scattered fraction is replaced by the formation of clusters from the dispersely fractured domains, and from this time the kinetics of the fracture process is determined by the interaction of the clusters that are foci of microscopic cracks [15]. The situation under consideration is analogous to that which exists in the theory of phase transitions [30], as well as in the mathematical theory of combustion and explosion [31]. As is known, the initial differential equations of the theory of combustion and explosion have continuous solutions, dependent in a continuous manner on the parameters, initial and boundary conditions. But spasmodicity of the solutions occurs in extracting the asymptotic, their criticality to a small change in the parameters; i.e., the nature of the solution changes substantially.

The growth of the microcrack bulk concentration is accompanied by an abrupt increase in the traceless component of the tensor p_{ik}. Inversion of the sign of p' (see Fig. 3b) is associated with the reflection of the compression wave from a free surface and the formation of a tension wave. As the amplitude of the initial load pulse increases, a transition occurs from the microcrack distribution with one maximum (Fig. 4a) to the multiple formation of foci (Fig. 4b), domains with a sharply increased rate of microcrack growth.

Experimental information about the regularities of cleavage fracture is contained indirectly in measurements of the velocity of the free surface of shock-loaded plates [32]. Displayed in Fig. 5 are computed velocity profiles of a free surface w (1 and 2 are rectangular and triangular pulses). The motion of the free surface is determined by interaction of the unloading and perturbation waves formed in the fracture zone as a result of microcrack growth. The emergence of the elastic predecessor with amplitude ~0.4 GPa is extracted on the w profiles. Because of the elastic predecessor, the emergence of the plastic shock and the subsequent damped fluctuations of the velocity w is observed during reverberation of the cleavage pulse. These fluctuations indicate that either a free surface or a domain with low dynamic stiffness appeared within the specimen [10]. The investigated regularities of the transition to macroscopic fracture permit giving an explanation of the overloading phenomenon during loading of microsecond durations and the associated ambiguity in determining the fracturing stresses [33]. Comparison of the curves $\sigma_{zz}(u_{zz})$ and $p_{zz}(u_{zz})$ (see Fig. 2) discloses a characteristic singularity, the difference in the maximal values of the stresses in the dependences $\sigma_{zz}(u_{zz})$ is not reflected in practice in the level of the critical values of the parameter p_{zz}.

The explanation of this situation is the existence of a self-similar solution for (1.5) and (1.7) that corresponds to the asymptotic on which the microcrack growth process is characterized by an explosive instability [13]. Therefore, the ambiguity in determining the fracturing stresses as well as the weak dependence of the time to fracture on the amplitude of the initial pulse (the phenomenon of the dynamic branch [8]) under loadings with duration $\sim 10^{-7}$ are related to the fact that under shock-loading conditions the crack formation process proceeds more rapidly than the growth of tensile stresses in the spall section, which indeed results in limiting of the overload [34].

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